The slope of a curve

The **slope** of a curve C at a point P is the slope of the tangent line to C at P if such a tangent line exists. In particular, the slope of the graph of $y = f(x)$ at the point x_0 is

$$\lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}.$$

In Exercises 1–12, find an equation of the straight line tangent to the given curve at the point indicated.

1. $y = 3x - 1$ at $(1, 2)$

2. $y = x/2$ at $(a, a/2)$

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$y = \frac{x}{2} = \frac{1}{2} \cdot x$

$y' = f'(x) = \frac{1}{2}$

$y' = \frac{1}{2} \cdot 1 = \frac{1}{2}$

f

3. $y = 2x^2 - 5$ at $(2, 3)$

x y

$y' = f'(x) = 4x$

$f'(2) = 4 \cdot 2 = 8 = m$

$y = mx + n$

$\downarrow$

$3 = \underbrace{8 \cdot 2}_{16} + n \Rightarrow n = -13$

$y = 8x - 13$

4. $y = 6 - x - x^2$ at $x = -2$

5. $y = x^3 + 8$ at $x = -2$

6. $y = \dfrac{1}{x^2 + 1}$ at $(0, 1)$

7. $y = \sqrt{x+1}$ at $x = 3$

8. $y = \dfrac{1}{\sqrt{x}}$ at $x = 9$

9. $y = \dfrac{2x}{x+2}$ at $x = 2$

10. $y = \sqrt{5 - x^2}$ at $x = 1$

11. $y = x^2$ at $x = x_0$

12. $y = \frac{1}{x}$ at $\left(a, \frac{1}{a}\right)$

21. Find all points on the curve $y = x^3 - x + 1$ where the tangent line is parallel to the line $y = 2x + 5$.

22. Find all points on the curve $y = 1/x$ where the tangent line is perpendicular to the line $y = 4x - 3$.

18. Find the slope of the curve $y = x^2 - 1$ at the point $x = x_0$. What is the equation of the tangent line to $y = x^2 - 1$ that has slope -3?

19. (a) Find the slope of $y = x^3$ at the point $x = a$.

(b) Find the equations of the straight lines having slope 3 that are tangent to $y = x^3$.

20. Find all points on the curve $y = x^3 - 3x$ where the tangent line is parallel to the x-axis.

16. $f(x) = |x^2 - 1|$ at $x = 1$

15. $f(x) = (x + 2)^{3/5}$ at $x = -2$